I. P. Castro & C. Vanderwel, *Turbulent Flows: An Introduction*, IOP, 2021.

Chapter 2 Sample Exercises

2.1 Derive the Reynolds equation (2.16) from the basic Navier-Stokes equation (2.4) using the Reynolds decomposition and anything else needed.

2.2 Derive equation (2.22) from (2.21) and show that if the turbulence is isotropic – i.e. if all statistics of the fluctuating velocity u_i' are not affected by coordinate system rotations or translations so that the normal stresses are equal – then the shear stress is necessarily zero.

2.3 Derive the Reynolds-averaged vorticity equation (2.23) from equation (2.12).

2.4 Derive the Reynolds stress transport equation (2.27).

2.5 Show that the full viscous term in the Reynolds stress equation (2.27) (ie. the viscous transport and the viscous dissipation) can also be written in the following forms:

$$\frac{\partial}{\partial x_k}\left(\nu\frac{\partial\overline{u_i'u_j'}}{\partial x_k}\right)-2\nu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}=\nu\overline{u_i'\frac{\partial^2 u_j'}{\partial x_k\partial x_k}}+\nu\overline{u_j'\frac{\partial^2 u_i'}{\partial x_k\partial x_k}}$$

2.6 Use the Reynolds stress transport equation (2.27) to derive the full turbulent kinetic energy (TKE) transport equation (2.28).

2.7 Show that the dissipation $\overline{\epsilon}$ as it appears in equation (2.28),

$$\overline{\epsilon}=\nu\overline{\frac{\partial u_i'}{\partial x_j}\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)},$$

can also be written equivalently as

$$\overline{\epsilon}=\frac{1}{2}\nu\overline{\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)}$$

or

$$\overline{\epsilon}=\nu\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}+\nu\frac{\partial^2\overline{u_i'u_j'}}{\partial x_i\partial x_j}.$$

Why is the second term on the right-hand side of this last relation identically zero in homogeneous turbulence?