I. P. Castro & C. Vanderwel, *Turbulent Flows: An Introduction*, IOP, 2021.

Chapter 4 Sample Exercises

4.1 Show that for $x(t) = a\sin(2\pi f t)$, an observation period T equal to any integral multiple of the period $1/f$ yields a probability density function of $x(t)$ given by equation (4.4).

4.2 Show that $\overline{u'^2} = \int_{-\infty}^{\infty}(u-U)^2 p(u)du$.

4.3 The file "TurbulenceSample.txt" contains data obtained by hot wire anemometry in a wind tunnel boundary layer. The streamwise velocity signal was sampled at 60 kHz for a total time of 30 seconds. Estimate the probability density function of the data, as defined by equation (4.3), by plotting the histogram of the data choosing appropriate bin sizes Δx.

4.4 Obtain the autocorrelation $R(\tau)$ for the turbulence signal provided by "TurbulenceSample.txt" and compare it with those for the white and pink noise signals in "NoiseSample.txt". Assuming Taylor's frozen flow hypothesis, apply equations (4.54) and (4.55) to calculate the integral length scale L_x and Taylor microscale λ of the turbulence.

4.5 The file "TurbulenceSample2.txt" contains the streamwise u, vertical v, and spanwise w components of velocity near the bottom wall of a channel flow from DNS.

a. Plot the joint probability density function of u/u_{rms} and v/v_{rms}, choosing appropriate bin sizes. A nice way to visualise this is to superimpose contours of the joint-pdf on top of a scatter plot of u/u_{rms} and v/v_{rms}. Does it imply a positive or negative correlation between the two components? The slope of the line of best fit should equal the correlation coefficient.

b. Determine the Reynolds shear stress (i) from the joint-pdf as in equation (4.35) and (ii) as the time-average of $u'v'$ from the data records.

4.6 Show that $E(f)$ for the first-order pink noise signal (i.e., having $R(\tau) = e^{-\lambda\tau}$) is given by equation (4.44). Find also the spectrum function for the second-order pink noise signal whose autocorrelation is given by equation (4.45).

4.7 Plot the one-dimensional energy spectrum of the velocity signal from the file "TurbulenceSample.txt". Explore how (i) the period and (ii) the sampling rate of the given turbulence signal affect the location of the first spectral estimate (i.e. $E(1)$) and the aliasing frequency. Explore also how splitting the total available signal into shorter batches can be used to improve the accuracy of spectral estimates. Compare this spectrum of a real turbulence signal with those computed for the white noise and pink noise signals provided in "NoiseSample.txt"