I. P. Castro & C. Vanderwel, *Turbulent Flows: An Introduction*, IOP, 2021.

Chapter 7 Sample Exercises

7.1. Derive the total stress equation for Couette flow given by (8.2) from the x-direction mean momentum equation. Are there any other flows where this would also be valid?

7.2. Assuming that turbulent Couette flow has an S-shaped velocity profile as sketched in Figure 8.2, sketch the expected shapes of the viscous stress (ie. $\mu dU/dy$), the turbulent stress (ie. $\rho\overline{u'v'}$), and the total stress (ie. τ) profiles over $0 < y/h < 2$. Compare your sketches with the results presented in figure 8.3.

7.3. For a turbulent channel flow, sketch the expected shapes of the viscous stress (ie. $\mu dU/dy$), the turbulent stress (ie. $\rho\overline{u'v'}$), and the total stress (ie. τ) profiles over $0 < y/h < 2$. Contrast these with those for Couette flow from the previous exercise.

7.4. The file "ChannelData.txt" contains the velocity profile within channel flow at Re_τ = 2,000 by Hoyas & Jiménez (2006). In this exercise we will look at how the velocity profile compares with the expected linear relationship in the viscous sublayer and the logarithmic relationship in the inertial sublayer.

a. Plot the velocity profile U^+ vs. y^+ in linear axes. Zoom in to the near wall region and compare the profile with $U^+ = y^+$ which we expect to see in the viscous sub-layer. Over what range of $y+$ is this valid?

b. Zoom out and adjust the axes to display this plot of U^+ vs. y^+ in log-linear axes. Fit the log law, equation (7.28), using $\kappa = 0.387$ and $A = 4.5$.

c. Plot the Kármán measure Π (given by equation (7.32)) vs. y^+. Confirm the approximate extent of the inertial sublayer and the value of $1/\kappa$ indicated by the plateau.

7.5. The file "PipeData.txt" contains velocity data in pipe flow at Re = 1,000,000 from the Princeton Superpipe by Zagarola & Smits (1997). In this exercise we will look at how the velocity profile compares with expected logarithmic and power law profiles.

a. Plot the velocity profile U^+ vs. y^+ in linear axes. Zoom in to the near wall region and compare the profile with $U^+ = y^+$ which we expect to see in the viscous sub-layer. Based on the location of the closest measurement to the wall estimate the extent (in physical units) of the viscous sublayer in this experiment.

b. Zoom out and adjust the axes to display this plot of U^+ vs. y^+ in log-linear axes. Fit the log law, equation (7.28), using $\kappa = 0.41$ and $A = 5.2$.

c. Adjust the axes to log-log scaling and consider whether a power law of the form $U^+ = C(y^+)^\gamma$ provides a better fit to the data in the near-wall overlap layer.