I. P. Castro & C. Vanderwel, *Turbulent Flows: An Introduction*, IOP, 2021.

Chapter 8 Sample Exercises

ERRATA: Please note there is a typo in the eBook in Question 8.2(a) in the expression for Re_θ which has been corrected below.

8.1. The file "TBLData.txt" contains the velocity profile of a smooth-wall turbulent boundary layer at $Re_\theta = 4{,}060$ from DNS by Schlatter and Orlu (2010). In this exercise we will look at how the velocity profile compares with expected viscous sublayer, log law, and wake profiles.

a. Plot the velocity profile U^+ vs. y^+ in linear axes. Zoom in to the near wall region and compare the profile with $U^+ = y^+$ which we expect to see in the viscous sub-layer. To what value of $y+$ is this valid? What percentage of the boundary layer depth (in terms of δ) does this region cover?

b. Zoom out and adjust the axes to display this plot of U^+ vs. y^+ in log-linear axes. Fit the log law eq (9.22) using $\kappa = 0.384$ and $A = 4.173$.

c. Plot the velocity profile against outer units (ie. U^+ vs. y/δ). Plot the log-law with the wake function (ie. eq. 9.27) with $\Pi = 0.55$. Note how this fits well in the outer region (and clearly doesn't apply beyond $y/\delta = 1$).

8.2. In this exercise we will consider the integral parameters that describe the velocity profile data given in the file "TBLData.txt". This file contains smooth-wall turbulent boundary layer at $Re_\theta = 4060$ from DNS by Schlatter and Orlu (2010).

a. Given that $c_f = 0.002971$ for this flow (and by definition, $u_\tau/U_\infty = \sqrt{c_f/2}$) integrate the velocity profile to confirm that the normalised displacement thickness is $Re_{\delta^*} = \int_0^\infty \left(\frac{U_\infty}{u_\tau} - U^+\right) dy^+ = 5633$, that the normalised momentum thickness is $Re_\theta = \frac{u_\tau}{U_\infty} \int_0^\infty U^+ \left(\frac{U_\infty}{u_\tau} - U^+\right) dy^+ = 4061$, and that $H = 1.387$.

b. Calculate the normalised value of the Rotta-Clauser integral thickness $Re_\Delta = \frac{U_\infty \Delta}{\nu} = \frac{U_\infty}{\nu} \delta^* \frac{U_\infty}{u_\tau} = Re_{\delta^*} \frac{U_\infty}{u_\tau}$. Determine the ratio of this with the normalised boundary layer thickness $Re_{\delta_{99}} = \frac{U_\infty \delta_{99}}{\nu} = \delta_{99}^+ \frac{U_\infty}{u_\tau}$ and compare this with the classical (constant) value of Δ/δ from the text.

8.3. This exercise explores the velocity profile over a rough surface and the challenges in determining u_τ and ΔU^+. The file "RoughWallData.txt" contains measurements of the velocity and Reynolds shear stress over a surface covered with 320 grit sandpaper in channel flow by Flack, Schultz, Barros & Kim (2016).

a. Try to fit a log-law in the form of equation (8.49) to the measurements of U versus y, by adjusting the values of u_τ, d, and $C = (A - \Delta U^+)$, assuming u_τ is unknown.

b. Plot the Reynolds stress and get a second estimate of u_τ from the peak of $-\overline{uv}$ in the inertial (log) layer as $-\overline{uv}_{peak} = u_\tau^2$. How does this compare with your previous estimate from the log law fit?

c. In these experiments, the wall shear stress was measured directly from the pressure drop in the channel and u_τ was found to be 0.155 m/s. How different were the previous estimates? Re-plot the log law in inner units and compare the results with a smooth wall channel flow (with kappa = 0.387, A = 4.5; ex 7.4) to determine ΔU^+.